Determination of Inertial Characteristics of a High Wing Unmanned Air Vehicle

A Halder, *Non-member*

V Agarwal, *Non-member*

R Garhwal, *Non-member*

Dr M Sinha, *Non-member*

To study the dynamics of an unmanned air vehicle (UAV), prior knowledge about the vehicle's inertial characteristics is essential. An experimental study and detailed analysis has been carried out to determine the centre of gravity (CG) and inertia tensor for a high wing UAV testbed. In this paper, an improved method has been suggested for aircraft inertial characterization in general, and for high wing low cost UAV in particular. To carry out the experiment, complete experimental set-up has been designed and fabricated in-house. The method proposed here, promises to provide superior estimation of inertial characteristics of high wing UAV, compared to the methods cited in the literatures.

Keywords **:** Inertia tensor; Centre of gravity; Inertia ellipse; UAV

NOTATION

A, B, C	: MI of the aircraft about the X body axis, Y body axis and Z body axis, respectively
A', B', C'	: MI of the aircraft about the first principal axis, second principal axis and third principal axis, respectively
l	: vertical distance of CG of aircraft from point of suspension
l'	: vertical distance of CG of cradle from point of suspension
I_G	: moment of inertia (MI) of swinging gear
m_a	: additional mass
T	: time period of oscillation of aircraft and swinging gear
T'	: time period of oscillation of swinging gear only
V	: volume of aircraft
w	: weight of the aircraft
w'	: weight of the swinging gear
ρ	: density of air

INTRODUCTION

The growing need to implement Intelligence, Surveillance and Reconnaissance (ISR) abilities in a low-cost flight vehicle, has lead to extensive research in various aspects of Unmanned Air Vehicles (UAV). Recent years have seen the birth and proliferation of a whole gamut of specialized researches focused on UAV, like vision based control, cooperative tracking and path planning, development of smart adaptive materials for structural health monitoring etc, which have augmented the classical research areas like aerodynamics, propulsion, structures and control. The inertial characteristics of an UAV have direct or far reaching consequences on each of these disciplines. Hence, it is imperative to determine the inertial characteristics of the vehicle even before the first flight.

A priori knowledge of the position of the CG and the inertia tensor, which together characterize inertial properties are crucial for sensor positioning, control law, and flight simulator design. The position of the CG can be estimated from weighing measurements while Moments of Inertia (MI) determination includes rough estimation from historical database, estimation using Computer-Aided-Design (CAD) software, and estimation by component build up method using the rule of vector addition. Although the lengthy and tedious calculation makes the last method almost prohibitive for a complex system like UAV, the other two methods are used in practice and are supplemented by experimental verification. This is necessary as these methods cannot capture the effect of air entrapped inside the airframe and the effect of additional mass of air moving with the airframe which is more significant for UAV as compare to an aircraft.

In 1926, spinning investigations first revealed the importance of experimental determination of MI of an airplane[1]. An experiment, based on pendulum method, was designed by National Advisory Committee for Aeronautics (NACA) for this purpose[2]. Later, this experiment was modified[3] to take into account the effects of entrapped and additional mass of air. In the modified experiment, rolling

A Halder, V Agarwal, R Garhwal and Dr M Sinha are with the Department of Aerospace Engineering, Indian Institute of Technology, Kharagpur 721 302.

This paper was received on April 22, 2008. Written discussion on the paper will be entertained till January 31, 2009.

and pitching MI (I_{xx} and I_{yy}) were determined by swinging the airplane as a compound pendulum and yawing MI (I_{zz}) was determined by swinging it as a bifilar torsional pendulum. For roll-yaw product of inertia (I_{zx}) determination, the nose up and nose down swinging were carried out in the *Z-X* symmetry plane in the compound pendulum set up. The accuracy of the MI obtained with the compound pendulum set up, was found to be highly sensitive to the measurement of suspension length[4] which in turn depends on the precision with which the vertical coordinate of the CG could be located. Plumb-line suspension method was initially adopted to find the vertical position of the CG. But experimenting with very high attitudes, as demanded by plumb-line suspension method, invited the danger of structural damage and uneasy handling.

The British recognized this problem and circumvented[5] this by swinging the aircraft for two different suspension lengths. Two simultaneous equations were then solved to find the unknown MI; but this could not find immediate application due to lack of knowledge about the effect of ambient air. The Russians successfully attempted the problem associated with plumb-line suspension method by developing a special kind of compound pendulum having two degrees-of-freedom[6]. This enabled the aircraft to perform two simultaneous oscillations in opposite directions about two different axes, and unlike the British method, the MI determination needed only one equation which was obtained directly about the aircraft axis. The complex nature of the oscillation hindered the widespread acceptance of this method.

In spite of some disadvantages, the methods described above work well for biplanes and parasol monoplanes. For low wing monoplanes, NACA developed a modified method[6] by employing the British proposal of using two different suspension lengths, formulating the effect of entrapped and additional mass of air. Although this method found wide acceptance for its simplicity and completeness, the need to compute the volume of the vehicle remained a significant source of error. For an UAV, volume computation is even more difficult due to heavy instrumentation. Furthermore, determination of MI for a high wing UAV has never been addressed in particular. The high wing configuration has enjoyed wide acceptance for the design of fixed wing UAV due to its ease of integration and less probability of wing damage, should there be a crash landing. This paper describes an attempt to compute the MI of a high wing UAV with a modification to NACA method for achieving better accuracy.

THEORY

This section elaborates the physics of the problem and provides the necessary mathematical formulation for finding the MI of a UAV.

Relation between Mass and Weight

Weight, as measured in the laboratory with the help of a spring balance, actually gives the net downward force acting on an object. For a block of mass *m* kept in atmosphere, there are two forces acting on the block the gravitational pull of the earth and the buoyant force due to atmosphere. The measured weight *w* will be

$$m = \frac{w}{g} + \rho V \tag{1}$$

Here *m* is the mass of the block; ρ, the density of air; *V*, the volume of the block and *w* is the weight as provided by the spring balance.

Effective Mass

A body moving in air carries with itself a layer of air as an envelope, therefore, imparts momentum to it. As a result, the effective mass of aircraft increases by some amount referred to as additional mass which depends upon the orientation of the aircraft relative to direction of motion, the velocity of aircraft, the density of air and the dimensions of the aircraft. Generally, for large density body this contribution is negligible but for lighter bodies, such as, aircraft, this contribution is significant especially for the UAV. In the case of aircraft, the value of this mass primarily depends on the aspect ratio of the wing. Detailed calculation of additional mass is given in earlier studies[3]. Thus, for an aircraft,

$$m_{\text{eff}} = m + m_a \tag{2}$$

$$m_{\text{eff}} = \frac{w}{g} + \rho V + m_a \tag{3}$$

Here m_{eff} is the effective mass of the aircraft and m_a, the additional mass of air.

Moment of Inertia

The true MI, *I* of an aircraft consists of three parts. A constant part, I_S representing the structural MI, I_E representing the MI of the entrapped air and varying with the density of air, and the MI, I_A of the air moving with the aircraft, *ie*, of the additional mass. Thus,

$$I = I_S + I_E + I_A \tag{4}$$

The experimental determination of *I*, which appears on the left side of equation (4), involves motion of the aircraft.

METHODOLOGY

The equation of motion of any pendulum, for small oscillation in vacuum, can be written as

$$I\frac{d^2\theta}{dt^2} + b\theta = 0 \tag{5}$$

Here *I*, is the MI of the pendulum about the axis of oscillation; b is a constant that depends on the dimensions and weight of the pendulum; and θ is the angular displacement of the pendulum. This gives the time period of oscillation as

$$T = \frac{2\pi}{\sqrt{\frac{b}{I}}} \tag{6}$$

which can be rearranged as

$$I = \frac{T^2 b}{4\pi^2} \tag{7}$$

For a compound pendulum, $b = wl$, where w is the weight of the pendulum and l, the distance of CG of pendulum from the axis of oscillation. Thus,

$$I = \frac{T^2 wl}{4\pi^2} \tag{8}$$

The MI about the axis passing through CG (parallel axis) is given as

$$I_{CG} = \frac{T^2 wl}{4\pi^2} - Ml^2 \tag{9}$$

Determination of Centre of Gravity

To determine the CG of the aircraft, the aircraft is placed on three weighing pans with each of three wheels resting on one of the pans. The reading of the pans directly gives the normal force acting on each of the wheels. If $\overline{F}$, $\overline{R}$ and $\overline{L}$ represent the normal reaction forces on the front, right and left wheels, respectively, then

$$x_{CG} = \frac{(\overline{L}h_1 + \overline{R}h_1)}{(\overline{F} + \overline{R} + \overline{L})} \tag{10}$$

$$y_{CG} = \frac{(\overline{R} - \overline{L})h_2}{(\overline{F} + \overline{R} + \overline{L})} \tag{11}$$

Here h_1 is the distance between the front landing gear and the left and right landing gear and h_2 is the half of the lateral separation between the left and right landing gear.

Determination of z_{CG} involves keeping the aircraft on an inclined plane in pitch down position. The forces acting on the aircraft are : (i) w, the weight of the aircraft, (ii) N_F, N_R and N_L, the reaction force normal to the inclined plane acting on the front, right and left wheels, respectively, (iii) f_F, f_R and f_L, the friction forces acting in the plane of inclination on the front, right and left wheels, respectively. Summation of moments about the axis passing through the rear wheels gives

$$z_{CG} = \frac{[N_F(h_1 \cos\theta) - w\cos\theta(h_1 - x_{CG})]}{(w \sin\theta)} \tag{12}$$

Here, $N_F \cos\theta$ can be measured by putting a weighing pan beneath the front wheel in a vertical position, and θ is computed using the relation

$$\theta = \sin^{-1}\left(\frac{h_3}{h_1}\right) \tag{13}$$

In equation (13), h_3 is the vertical distance between the front and the rear wheels while resting on the weighing pans. It was observed that z_{CG} was very sensitive to the value of θ, therefore, a high value of θ was chosen for which $\frac{\partial z_{CG}}{\partial \theta}$ was small in magnitude. The CG was found close to the rear wheel. Hence, the pitch down position was used as this position gave the liberty of using high θ in the experiment.

Determination of Moment of Inertia

The determination of the MI of aircraft is done by pacing it on the swinging gear and oscillating it as a pendulum to avoid suspension about any point on the aircraft structure. By noting the time period of oscillation, the MI of this assembly about the axis of oscillation is calculated. The MI of the swinging gear alone is determined by swinging it separately and then is subtracted from the MI of the aircraft and swinging gear system which gives the MI of the aircraft alone about the axis of oscillation. To find the MI about an axis passing through CG, parallel axis theorem is used.

Moment of Inertia about the X-body Axis

To determine the MI about X-body axis, the aircraft is swung[3] as shown in Figure 1.

The MI, A will be given by,

$$A = \frac{T^2 wl}{4\pi^2} - \left(\frac{w}{g} + \rho V + m_a\right)l^2 - I_G \tag{14}$$

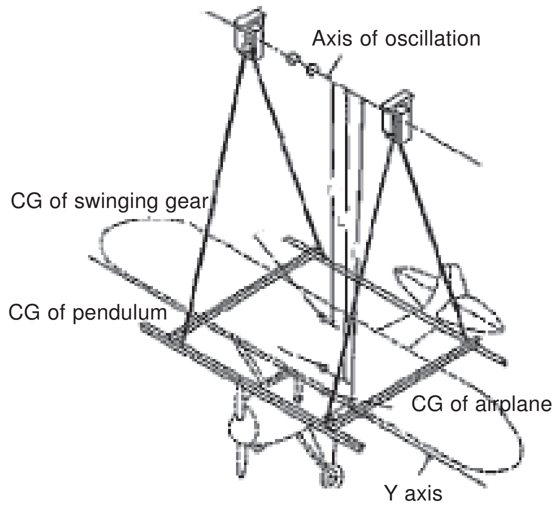

Figure 1 Determination of MI about the Y body axis (to determine about the X body axis, aircraft is rotated by 90° on the cradle[3])

The first term on the right hand side gives the MI of the aircraft and swinging gear assembly about the axis of oscillation. I_G, denotes the MI of swinging gear only, about the axis of oscillation and is determined by swinging the gear as an independent pendulum. The term $\left(\frac{w}{g} + \rho V + m_a\right) l^2$ is due to the application of the parallel axis theorem. If the time period of swinging gear is T' then I_G will be given by,

$$I_G = \frac{T'^2 l' w'}{4\pi^2} \tag{15}$$

Using principle of moments,

$$wl = wl + w'l' \tag{16}$$

Substituting the value of *wl* [equation (16)] into equation (14), following equation is obtained

$$A = \frac{T^2 (wl + w'l')}{4\pi^2} - \left(\frac{w}{g} + \rho V + m_a\right) l^2 - I_G \tag{17}$$

l' is obtained by using the geometry of the cradle; T and T' are known experimentally. Since, l' and T' are known, I_G can be obtained using equation (15). In equation (17), l, A, and $(\rho V + m_a)$ are the unknowns. To solve these unknowns, three equations would be required. Hence, equation (17) is written for three lengths (denoted by subscripts 1, 2 and 3) as follows.

$$A = \frac{T_1^2 (w l_1 + w' l'_1)}{4\pi^2} - \left(\frac{w}{g} + \rho V + m_a\right) l_1^2 - I_{G1} \tag{18}$$

$$A = \frac{T_2^2 (w l_2 + w' l'_2)}{4\pi^2} - \left(\frac{w}{g} + \rho V + m_a\right) l_2^2 - I_{G2} \tag{19}$$

$$A = \frac{T_3^2 (w l_3 + w' l'_3)}{4\pi^2} - \left(\frac{w}{g} + \rho V + m_a\right) l_3^2 - I_{G3} \tag{20}$$

l_2 and l_3 are related to l_1 by

$$l_2 = l_1 + \Delta l_2 = l_1 + (l'_2 - l'_1) \tag{21}$$

$$l_3 = l_1 + \Delta l_3 = l_1 + (l'_3 - l'_1) \tag{22}$$

Equations (21) and (22) are substituted in equations (19) and (20), respectively, to get

$$A = \frac{T_2^2 (w(l_1 + (l'_2 - l'_1)) + w' l'_2)}{4\pi^2} - \left(\frac{w}{g} + \rho V + m_a\right)(l_1 + (l'_2 - l'_1))^2 - I_{G2} \tag{23}$$

$$A = \frac{T_3^2 (w(l_1 + (l'_3 - l'_1)) + w' l'_3)}{4\pi^2} - \left(\frac{w}{g} + \rho V + m_a\right)(l_1 + (l'_3 - l'_1))^2 - I_{G3} \tag{24}$$

In equations (18), (23) and (24), the unknowns are A, l_1 and $\frac{w}{g} + \rho V + m_a$. Thus, these equations can be solved to determine the unknowns. This method was initially chosen to find A. But, it was later found that even the slightest experimental error renders the above system of equations inconsistent. Thus, as an alternative approach, l_1 was determined by other means and substituted in equations (21) and (22) to give l_2 and l_3. Equations (18), (19) and (20) now become three equations in two variables. Thus, choosing two equations at a time, three values of A, *ie*, A_1, A_2 and A_3 were obtained and then the average of these three values was taken. Solving equations (18) and (19), A_1 is obtained

$$A_1 = \frac{[T_1^2 (w l_1 + w' l'_1) l_2^2 - T_2^2 (w l_2 + w' l'_2) l_1^2]}{[4\pi^2 (l_2^2 - l_1^2)]} + \frac{I_{G2} l_1^2 - I_{G1} l_2^2}{(l_2^2 - l_1^2)} \tag{25}$$

Solving equations (19) and (20), A_2 is obtained

$$A_2 = \frac{[T_1^2 (w l_1 + w' l'_1) l_3^2 - T_3^2 (w l_3 + w' l'_3) l_1^2]}{[4\pi^2 (l_3^2 - l_1^2)]} + \frac{I_{G3} l_1^2 - I_{G1} l_3^2}{(l_3^2 - l_1^2)} \tag{26}$$

Solving equations (18) and (20), A_3 is obtained

$$A_3 = \frac{[T_2^2 (w l_2 + w' l'_2) l_3^2 - T_3^2 (w l_3 + w' l'_3) l_2^2]}{[4\pi^2 (l_3^2 - l_2^2)]} + \frac{I_{G3} l_2^2 - I_{G2} l_3^2}{(l_3^2 - l_2^2)} \tag{27}$$

$$A = \frac{(A_1 + A_2 + A_3)}{3} \tag{28}$$

Moment of Inertia about Y-body Axis

The CG of the aircraft is known. Thus, aircraft is swung at two different lengths about an axis parallel to the Y-body axis and equations analogous to equations (18) and (19) are written and then solved.

$$B = \frac{T_1^2(wl_1 + w'l_1')}{4\pi^2} - \left(\frac{w}{g} + \rho V + m_a\right) l_1^2 - I_{G1} \tag{29}$$

$$B = \frac{T_2^2(wl_2 + w'l_2')}{4\pi^2} l - \left(\frac{w}{g} + \rho V + m_a\right) l_2^2 - I_{G2} \tag{30}$$

$$B = \frac{[T_1^2(wl_1 + w'l_1')l_2^2 - T_2^2(wl_2 + w'l_2')l_1^2]}{[4\pi^2(l_2^2 - l_1^2)]} + \frac{I_{G2}l_1^2 - I_{G1}l_2^2}{(l_2^2 - l_1^2)} \tag{31}$$

Moment of Inertia about the Z-body Axis

To determine MI about Z-body axis, C, the aircraft and the swinging gear assembly is swung as a bifilar torsional pendulum about a vertical axis passing through the CG of the aircraft. Here, it is to be made sure that the CG of the aircraft and the swinging gear lie on the same vertical line[3] as shown in Figure 2. Noting the time period and using equation (21), the MI of the aircraft and swinging gear combined is obtained. The swinging gear is then swung as an independent pendulum, and again using equation (21), its MI is obtained. Thus,

$$C = \frac{wA^2T^2}{16\pi^2 d} - \frac{w'A^2T'^2}{16\pi^2 d} \tag{32}$$

It is to be noted here that w' in equation (32) is different from the earlier w' notation used. This is because of the spacer, which increases the weight of the swinging gear.

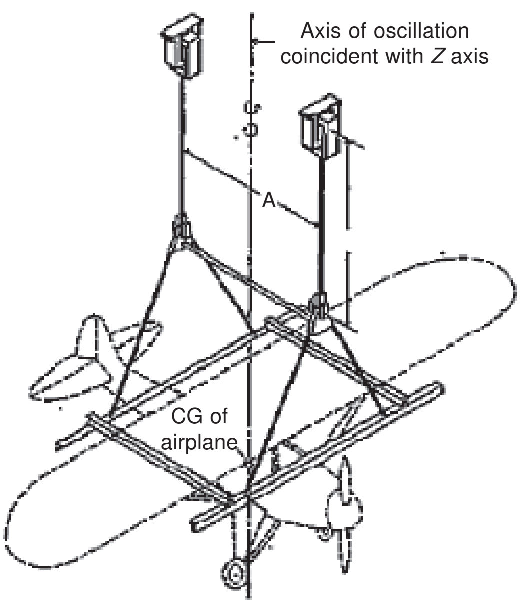

Figure 2 Determination of MI about the Z-body axis[3]

Principal Axes

In practice, for an aircraft the body axes approximate the principal axes being close to it, but for some applications a precise estimate of the principal axes may be desirable. XZ-plane being a plane of symmetry, the Y-axis forms one of the principal axes. The other two lie in the XZ-plane and need to be located. If τ is the angle between the X-body axis and one of the principal axis then τ can be written as

$$\tau = \frac{1}{2}\tan^{-1}\left(\frac{2D}{(C - A)}\right) \tag{33}$$

The product of inertia D in above equation is found using,

$$D = \frac{(A\cos^2\theta + C\sin^2\theta - E)}{\sin 2\theta} \tag{34}$$

Here E is the MI of the aircraft about an axis inclined at θ to the X-axis and passing through the CG. E, is found experimentally by swinging the aircraft about an axis passing through the XZ-plane and inclined at an angle θ to the X-body axis. Equation (17) is employed and $\frac{w}{g} + \rho V + m_a$ is substituted from the value obtained during the simultaneous solution of equations (18) and (19) to get B. I_G is evaluated by swinging the gear alone at the same angle θ and at the same length l' and using equation (15).

$$E = \frac{T^2(wl + w'l')}{4\pi^2} - \left(\frac{w}{g} + \rho V + m_a\right) l^2 - I_G \tag{35}$$

The MI about the principal axes, A',B' and C' are now obtained using

$$\left.\begin{aligned} A' &= A\cos^2\tau + C\sin^2\tau + D\sin 2\tau \\ B' &= B \\ C' &= A\sin^2\tau + C\cos^2\tau - D\sin 2\tau \end{aligned}\right\} \tag{36}$$

EXPERIMENTAL SET-UP COMPONENTS

The experimental set-up consists of a number of components, such as, swinging gear, knife edge to provide a definite axis of oscillation[3] (Figure 3), spacer rod for the bifilar torsion assembly, universal joint to ensure that the pendulum CG always remains in the same vertical line as the axis of oscillation[7] (Figure 4), cradle to keep the aircraft for swinging and is made up of two 'I' beams and two 'L' beams. This assembly was chosen because it offers minimum damping when moving in air.

RESULTS

Measuring from the centre of the front wheel, the CG position was found to be (x_{CG}, y_{CG}, z_{CG}) = (58.1 cm,

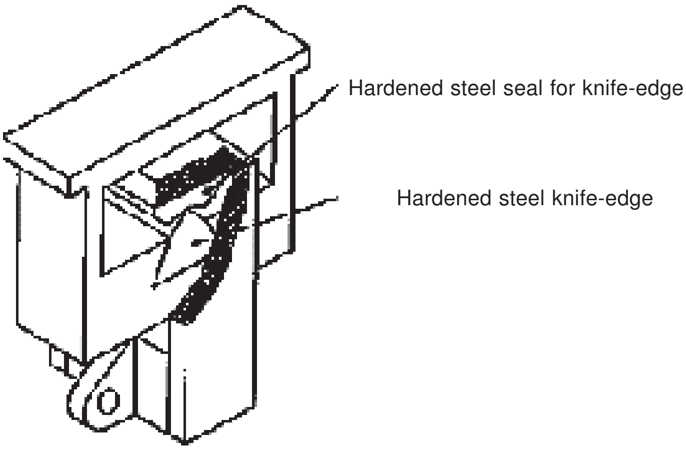

Figure 3 Knife edge[3]

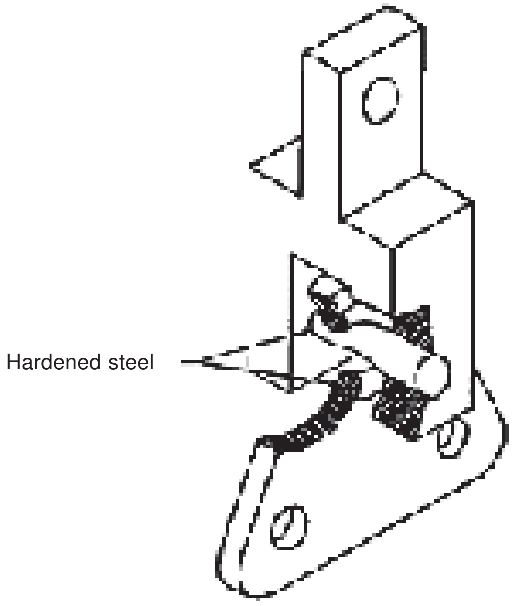

Figure 4 Universal joint[3]

0.3 cm, and 41 cm). Using z_{CG} measurement, it was found that I_{xx} = 3.783 kgm^2, I_{yy} = 3.76 kgm^2, I_{zz} = 6.928 kgm^2 and I_{zx} = –1.48 kgm^2. The principal MI were also computed: I'_{xx} = 5.223 kgm^2, I'_{yy} = 3.76 kgm^2 and I'_{zz} = 5.486 kgm^2. The inclination (τ) between body axes and principal axes was calculated, $\tau = -21.63°$.

A detailed error analysis was carried out for this experiment. It was found that the aerodynamic damping ratio was 0.0113. The frictional effect of the support equipments was minimized by manufacturing them using high quality mild steel and hardened steel with CNC machining. The non-linear effect of gravity moment during roll oscillations was treated according to earlier study[8]. The experimental accuracy was found to be sensitive on Steiner's term. It was concluded that experiments with shorter suspension lengths produce less error.

CONCLUSION

A new method for determining MI of an aircraft has been presented which is particularly suitable for UAV with high-wing configuration. The novelty of the method is twofold. First, unlike previous methods, it obviates the volume computation of the vehicle, which is a major source of error. Secondly, three different short suspension length measurements were used instead of two, as a conscious effort to reduce the error due to suspension length(s).

REFERENCES

1. M W Green. 'Measurement of the Moments of Inertia of Full Scale Airplanes'. *NACA Technical Note No 265*, 1927.

2. M P Miller. 'An Accurate Method for Measuring the Moments of Inertia of Airplanes'. *NACA Technical Note No 351*, 1930.

3. H A Soulé and M P Miller. 'The Experimental Determination of the Moments of Inertia of Airplanes'. *NACA Report No 467*, 1933, p 501.

4. W Gracey. 'The Experimental Determination of the Moments of Inertia of Airplanes by Simplified Compound-pendulum Method'. *NACA Technical Note No 1629*, 1948, p 1.

5. S B Gates. 'The Determination of the Moments of Inertia of Aeroplanes'. *R &M No 1415*, *British A R C*, 1932.

6. J Pobedonoszeff. 'An Experimental Method of Determination of the Moments of Inertia of Aeroplanes'. *Report No 201*, *Transactions on CAHI*, Moscow, 1935.

7. J J A Gomez. 'Experimental Methods for Determining Moments of Inertia of Small UAVs'. Masters Thesis, *LiTH-IKP-EX-1903*, *Department of Mechanical Engineering*, *IKP*, *Linköping University*, 2001, p 1.

8. C H Wolowicz and R B Yancey. 'Experimental Determination of Airplane Mass andInertial Characteristics'. *NASA Technical Report*, *NASA TR R-433*, 1974, p 1.