FLIER: A Novel Sensor Fusion Algorithm

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***Abstract*- This paper proposes a novel sensor fusion algorithm to obtain instantaneous position and attitude estimates, which can either be used for aerial navigation or can be utilized to construct state feedbacks for camera stabilization. A divergence control strategy has also been formulated and the algorithm was embedded in real-time hardware. A comparative study between the proposed and conventional algorithm illustrates its efficacy.**

***Keywords*-Sensor fusion, Kalman filter, unmanned air vehicles (UAVs), hardware-in-loop-simulation (HILS).**

I. INTRODUCTION

This has been a widely recognized [1] fact that while solving an engineering problem, hardware implementation can throw light in areas where theory and simulation lack. Many engineering problems involve constraints like synchronization, computational complexity, size, weight, power, cost, saturation etc. which are either knowingly neglected in theoretical description for the sake of mathematical simplicity or are too resource-demanding to run a meaningful simulation. In addition to assess the credibility of the theory itself, hardware experiments also provide an avenue to testify the assumptions which lay the foundation of the theory. With this motivation, an attempt has been made to implement Fuzzy Logic Induced Extended Kalman Filter (FLIER): a novel sensor fusion algorithm in actual hardware for the purpose of real-time position and attitude estimation of an unmanned air vehicle (UAV) to aid its navigation.

The state estimates obtained from the embedded sensor fusion algorithm proposed in this paper, can also be used to construct artificial sensor feedbacks for pan-tilt stabilization of the camera and sonar altimeter platform affixed to the UAV. The aim of this work was to develop a robust sensor fusion algorithm to estimate the spatial positions and angular orientations of the UAV at any time instant and to implement it in real-time custom-made on-board hardware. The problem under consideration could be divided into the following sub-problems:

(a) Development of a basic sensor fusion algorithm which can fuse the data from inertial measurement unit (IMU) and global positioning system (GPS) to compute the instantaneous position coordinates of the UAV measured from the inertial coordinate system with flat earth assumption (x, y, z) and the instantaneous bank angle, elevation angle and heading or azimuth angle of the UAV (ϕ, θ, ψ) respectively.

(b) Development of a suitable divergence control strategy to guarantee the robustness of the sensor fusion algorithm, which can eliminate the divergence due to finite precision arithmetic of the processor.

(c) The last but not the least task was to embed the entire algorithm in the on-board hardware and to test it in real time i.e., hardware-in-loop-simulation (HILS).

There are various sensors interfaced with the on-board microcontroller of the UAV to acquire flight data. The data coming from these sensors are, in general, noise corrupted. One can expect all kinds of complexities in the measurements like the presence of multiple sensors for some variables (e.g., total velocity), absence of sensors for some variables (e.g., attitude) and even if, unique sensors are available for some variables, they are noise corrupted. Hence, it is imperative to have an algorithm which can yield realistic, unique and optimal values of the desired quantities describing the state of the system (in this case, positions and attitudes) by fusing the data coming out from various sensors. Such an algorithm is termed as sensor fusion algorithm.

A UAV poses several unique constraints which make the implementation of sensor fusion algorithm a challenging task. These include space constraint, weight constraint, power constraint and budgetary constraint. So the economics of the problem demands a solution which is smaller, cheaper and faster. This paper proposes such an algorithm and demonstrates the efficacy of it through real-time hardware-in-loop-simulation.

The organization of this paper is as follows. Section I served as an introduction. Section II sketches the literature survey. Section III discusses the method of attitude measurement. Section IV describes the sensor fusion algorithm with extended Kalman filter (EKF) implementation. Section V illustrates the FLIER algorithm and addresses the robustness issue by employing a divergence control strategy. Hardware architecture is described in Section VI. Section VII gives the results of real-time hardware implementation and demonstrates the superiority of the FLIER algorithm over the conventional EKF implementation. Section VIII concludes the paper.

II. Literature Survey

While Schmidt [2] described the computations for a gimbaled Inertial Navigation System (INS) with 9 state Kalman filter, Grewal and Andrews [3, 4] gave a model for a Kalman filter with 54 states.

Wolf *et al.* [5] have implemented a 27 state Kalman filter. Grejner-Brzezinska *et al.* [6] used a 21 state Kalman filter and an order of accuracy of 10 cm was achieved.

Srikumar and Deori [7] used an air-data based dead reckoning system for the UAV Nishant. Randle and Horton [8] described a 23 state Kalman filter. A 32 state Kalman filter was proposed by Brown and Sullivan [9] by integrating IMU with GPS using InterNav software made by Navsys Corporation [10] using a quaternion integration algorithm [11]. Moon *et al.* [12] processed the GPS values before sending it to their 9 state Kalman filter. The combined INS/GPS showed very good performance for recovering the gravity signal.

Mayhew [13] proposed several methods for improving position estimates of a system by incorporating fuzzy based sensor fusion techniques and a map-matching algorithm with 9 state Kalman filter. Moore and Qi [14] have implemented a 8 state direct Kalman filtering technique to integrate their INS and GPS. They used two stage filtering to preprocess the GPS data before the Kalman filter can use it.

Panzieri *et al.* [15], and Dorobantu and Zebhauser [16] used 5 state EKF for a land vehicle (2D case). Niculescu [17] applied various sensor fusion algorithms for the simplified case. The unscented Kalman filter (UKF) developed by Simon Julier *et al.* [18, 19] and its variant square root UKF developed by Brunke [20] and Van der Merwe and Wan [21] was implemented by Niculescu to yield superior performance.

The past research in aircraft sensor fusion algorithm has primarily strived for better accuracy, either by incorporating sophisticated hardware to improve arithmetic precision or by implementing a filtering scheme with higher algorithmic complexity. Neither of these approaches lead to a cost-effective solution from system integration point of view. The main contribution of this paper lies in designing a novel sensor fusion algorithm taking sensor uncertainty and ambiguity into account in a way that is realizable in low-cost hardware.

III. Attitude Measurement

To estimate the states using sensor fusion algorithm, one would need its measurements. But direct sensor measurement of attitudes is not easy. So attitude measurements need to be formulated using other directly measurable variables. One method is to use the carrier phase of the GPS signals [22, 23]. This involves at least three GPS antennas with a known geometry. Once the phase ambiguity is resolved, phase differences between the antennas can be calculated and a good estimate of attitude is made. The solution improves as the baseline between the antennas increases, which is difficult to achieve in small UAVs. Gebre-Egziabher *et al.* [24] proposed an ultra short baseline solution (approx 36 cm baseline), but even this is too large and heavy for small UAVs [25]. Another promising method is to use vector measurements of the magnetic and gravitational fields and then solve a set of nonlinear equations using optimization methods to come up with an attitude measurement [26, 27]. Akella *et al.* [28] formulated a feedback law that directly regulates the attitude with only gyro and inclinometer measurements.

In this paper, attitude measurements are constructed as follows:

(a) Obtain three consecutive GPS position measurements,
(b) difference the three GPS measurements to obtain two velocity measurements,
(c) average the two velocity measurements to give average velocity over 2 seconds,
(d) calculate the heading ψ from velocity as, $\psi = \tan^{-1}\left(\frac{\dot{Y}}{\dot{X}}\right)$
(e) difference the GPS calculated velocities to obtain a GPS acceleration measurements a_{GPS},
(f) average the accelerometers over the same 2 seconds as the GPS velocity is calculated to obtain a,
(g) calculate θ and ϕ using the accelerometers and the GPS acceleration rotated by ψ from corresponding equations [29].

IV. Sensor Fusion Using Extended Kalman Filter

This section gives a summary of attitude filtering and position filtering respectively using EKF. In the attitude filtering algorithm, A_k and C_k are the Jacobians of the dynamics and measurements respectively.

A. Attitude filtering using EKF:

Initialize with

$$\hat{\vec{x}}_0 = \mathrm{E}[\vec{x}_0] \quad (1)$$

$$\vec{P}_0 = \mathrm{E}[(\vec{x}_0 - \hat{\vec{x}}_0)(\vec{x}_0 - \hat{\vec{x}}_0)^T] \quad (2)$$

Time Update

$$\hat{\vec{q}}_{k+1} = \hat{\vec{q}}_k + T_{sampling}\Omega(\vec{\omega}_k - \hat{\vec{b}}_k)\hat{\vec{q}}_k \quad (3)$$

$$\hat{\vec{q}}_{k+1} = \frac{\hat{\vec{q}}_{k+1}}{\|\hat{\vec{q}}_{k+1}\|} \quad (4)$$

$$\hat{\vec{b}}_{k+1} = \hat{\vec{b}}_k \quad (5)$$

$$P_{k+1} = P_k + T_{sampling}(A_k P_k + P_k A_k^T + Q) \quad (6)$$

Measurement Update

$$K_k = P_k C_k^T (C_k P_k C_k^T + R)^{-1} \quad (7)$$

$$\hat{\vec{x}}_{k+1} = \hat{\vec{x}}_k + K_k(z_k - Euler(\hat{\vec{q}}_k)) \quad (8)$$

$$\hat{\vec{q}}_{k+1} = \frac{\hat{\vec{q}}_{k+1}}{\|\hat{\vec{q}}_{k+1}\|} \quad (9)$$

$$P_{k+1} = (I - K_k C_k)P_k \quad (10)$$

B. Position filtering using EKF :

State

$$\bar{x} = \begin{bmatrix} x & y & z \end{bmatrix}^T \tag{11}$$

Time Update

$$\hat{\bar{x}}_{k+1} = \hat{\bar{x}}_k + T_{samplins}\left(DCM^T \begin{bmatrix} V_P \\ 0 \\ 0 \end{bmatrix} \right) \tag{12}$$

$$P_{k+1} = P_k + T_{sampling}\,(G_k Q G_k^T) \tag{13}$$

Measurement Update

$$K_k = P_k (P_k + R)^{-1} \tag{14}$$

$$\hat{\bar{x}}_{k+1} = \hat{\bar{x}}_k + K_k(\bar{z}_k - \hat{\bar{x}}_k) \tag{15}$$

$$P_{k+1} = (I - K_k)P_k \tag{16}$$

All the symbols have their usual meanings [3]. Here G_k is the partial derivative of the dynamics with respect to each of the inputs. Formulation of G_k can be found in [29].

V. Divergence Control And FLIER Algorithm

The EKF code for attitude and position filtering may diverge because of the errors in modeling the system, finite precision arithmetic and associated truncation/round-off errors, which can be potentially dangerous. Many classical and ad hoc methods (like artificial noise injection) have been proposed to combat the divergence problem.

A detailed account on the causes of Kalman filter divergence and various divergence control strategies can be found in [30] along with the outline of FLIER algorithm. The objective of this algorithm is to guarantee the positive definiteness of the state covariance matrix (P) by suitably formulating the process noise covariance matrix (Q). The process noise covariance matrix (Q), in such a case, drives the state covariance matrix in a way that ensures the convergence of EKF.

A. Formulation of process noise covariance matrix:

To facilitate computation, it's a common practice to diagonalize Q, which has been proposed to be formulated as

$$Q_{ij} = |\Delta\bar{x}\Delta\dot{\bar{x}}|\,\Delta t\,\delta_{ij} \tag{17}$$

where $\Delta\bar{x}$ is the state residual and $\Delta\dot{\bar{x}}$ is the derivative of the state residual. δ_{ij} is the Kronecker delta. In order to calculate the derivative of the state residuals, the following method has been proposed by the authors.

B. Proposed FLIER Algorithm:

A linear fuzzy regression model with suitable size of the moving window was adopted to plot the state residue curve. On a sample data set, this provided better result than least square plot with moving window [30]. So the linear fuzzy regression with moving window was applied for the case of non-fuzzy data. In this algorithm, the state residual vector is formulated using linear fuzzy regression as

$$\Delta\bar{x} = f(t,\widehat{A}) = \widehat{A}_0 + \widehat{A}_1 t_1 + \widehat{A}_2 t_2 + \ldots + \widehat{A}_n t_n \tag{18}$$

where $\widehat{A}_i$ is the i^{th} fuzzy coefficient, a fuzzy number (contrary to conventional crisp constant). Since each fuzzy number can be characterized by its membership function, each $\widehat{A}_i$ was expressed as an isosceles triangular membership function such that c_i is the spread (half-width of the base) and p_i is the mid point of the base. Thus the aim of the fuzzy regression problem considered here is to determine a family of such symmetric triangles representing all the coefficients in the linear fuzzy regression formula.

It can be shown that [30], the corresponding symmetric triangular fuzzy membership function for the fuzzified state residual vector has mid-point and spread of $\sum_{i=1}^{n} p_i t_i$ and $\sum_{i=1}^{n} c_i\,|\,t_i\,|$, respectively.

Now one needs to find the fuzzy coefficients such that the spread of the fuzzy output is minimized. Tanaka *et al.* [31] formulated the objective function for this optimization problem as

$$O_f = \min\left\{ mc_0 + \sum_{j=1}^{m}\sum_{i=1}^{n} c_i t_{ij} \right\} \tag{19}$$

where $t_{0j} = 1 \forall j = 1,2,\ldots,m$ ... This optimization problem can be viewed as the minimization of total fuzziness (hence ambiguity) of the fuzzy linear model. This objective function O_f needs to be minimized subjected to two inequality constraints (derived by Tanaka *et al.* [31])

$$x_j \geq \sum_{i=0}^{n} p_i t_{ij} - (1-h)\sum_{i=0}^{n} c_i t_{ij} \tag{20}$$

$$x_j \leq \sum_{i=0}^{n} p_i t_{ij} + (1-h)\sum_{i=0}^{n} c_i t_{ij} \tag{21}$$

Equation (20) and (21) represents total $2m$ constraints. Now this becomes a linear programming problem, which must be solved to find the mid points and spreads of the fuzzy coefficients. The authors used Simplex method to solve this constrained optimization problem.

VI. Hardware Architecture

The hardware platform consists of two printed circuit boards (PCBs), viz. sensor interface card and processor card arranged in a dual stack configuration connected by 88 pins. The upper PCB is the sensor interface card with which takes the sensor outputs and sends them to the lower PCB i.e., processor card. In addition to interfacing the sensors, sensor interface card also synchronizes the bit rate of all sensors to the processing speed of the microcontroller (9600 bps).

The 32 bit microcontroller used in this custom-designed board is Motorola MC68332. Its features include a 32 bit CPU (CPU32), a system integration module (SIM), a time processor unit (TPU), a queued serial module (QSM), 2 kB static RAM with TPU emulation capability (TPURAM), maximum system clock speed of 20.97 MHz and high density complementary metal-oxide semiconductor (HCMOS) architecture to make low power consumption of the MCU.

Among various sensors interfaced with the processor card (via sensor interface card), IMU and GPS are of particular interest as FLIER fuses data coming from the two. The IMU consists of three gyros (ADXRS150) and three accelerometers

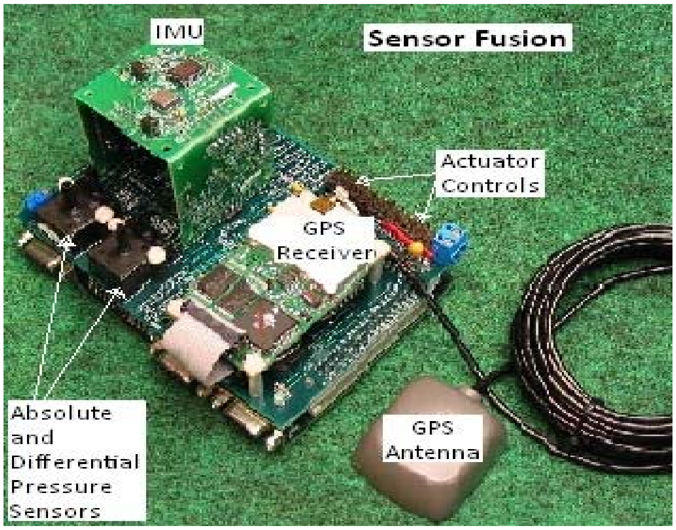

Fig. 1. Sensor Interface Card and Processor Card with Sensors

(ADXL105). The gyros measure body angular velocities and the accelerometers measure translational accelerations. The GPS gives spatial position of the board.

VII. Hardware In Loop Simulation

In this actual hardware implementation, driving noise covariance matrix Q has been formulated with and without FLIER (using random noise in the same range). The comparative results for real-time runs bring forth the superiority of FLIER.

A. *Hardware specific consideration:*

Since measurement update occurs once in a second (GPS frequency is 1 Hz.), so in order to make better estimate, the conventional once time update, then only measurement update – philosophy was modified. In fact, time update (predictor step) propagates for 3 time steps, then one measurement update is done i.e., measurement update was done at the fourth time step (since then only measurements are available). This improved the result.

Also care must be taken while implementing matrix inversion since it is memory consuming, and for computing trigonometric (or inverse trigonometric) functions (like arctan) complexity issues must be addressed. An intentional effort was made to reduce the number of nested loops in the code. Last but not the least, one should be cautious about the units while employing transformations between Euler angles and quaternions.

One incident that taught us a lesson was when the code kept diverging for few days even after our careful attempts. Later it was discovered that if the data starts queing in the microcontroller and its size exceeds that of the RAM, only the most recent entries were entered automatically by overwriting the earlier data. This possibly illustrates that in such an application, one has to be extremely careful about both time and space complexity.

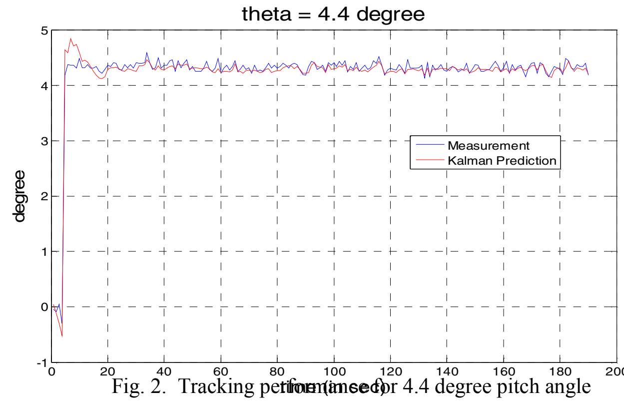

Fig. 2. Tracking performance for 4.4 degree pitch angle

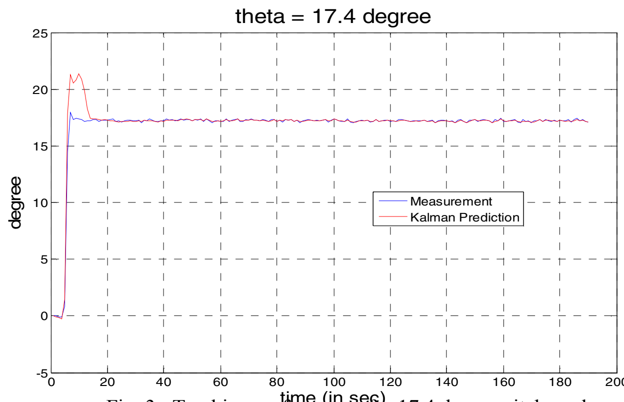

Fig. 3. Tracking performance for 17.4 degree pitch angle

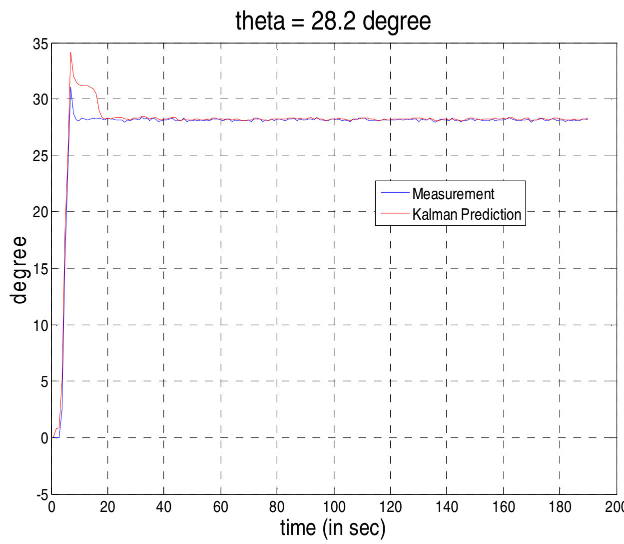

Fig. 4. Tracking performance for 28.2 degree pitch angle

B. *The effect of the magnitude of attitude on tracking performance:*

For a large set of pitch angles, tracking performance was compared and the performance was found to be satisfactory (less than 1% error). In evaluating tracking performance, reference measurement of attitude was assumed to be the constructed measurement as explained in section III. Fig. 2, 3, and 4 plots the tracking performance for three different pitch angles, viz. 4.4 degrees, 17.4 degrees and 28.2 degrees.

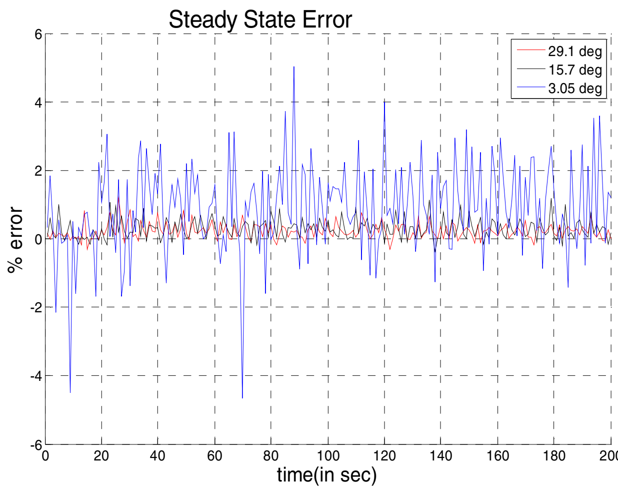

Fig. 5. Steady state error comparisons for three different pitch angles

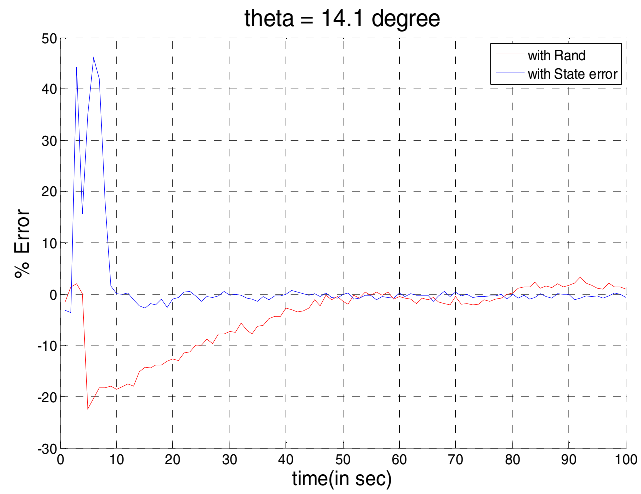

Fig. 6. Effect of driving noise for 14.1 degree pitch angle

So it can be concluded from the above plots that the magnitude of the angle does not have significant effect as far as tracking performance is concerned. However, when steady state errors for three different pitch angles were plotted together, it turns out that smaller the magnitude of the angle, larger will be the deviations from the steady state value. Fig. 5 illustrates this fact (three representative pitch angles were chosen to be 29.1 degrees, 15.7 degrees and 3.05 degrees).

C. The effect of the driving noise

The objective of this study was to investigate how the formulation of process noise covariance matrix (Q) affects the performance of the filter. So for three different pitch angles (14.1 degrees, 35.1 degrees and 44.6 degrees) FLIER code (with adaptive fuzzy state noise driven EKF) and conventional EKF code (with square of the random numbers as the entries of the diagonal Q matrix) were run simultaneously. The results are plotted in Fig. 6, 7, and 8, where the percentage estimation errors (with respect to the measurements) were plotted against time.

Some general observations can be made from Fig. 6, 7, and 8. It can be noted that the maximum overshoot in the transient state (corresponding to changing the angle) for state error formulation is much larger compared to that with random noise. However, the settling time is significantly larger for the case when random noise is used. Also, as it's evident from the last three plots, random noise formulation has slight tendency of overshoot in steady state too (i.e., the steady state error value for random noise is higher). It can be noted that, in all three plots above, the random noise curve slowly captures the state noise curve from bottom and has a tendency to cross it to a positive value.

To investigate it further, steady state errors (after convergence) were magnified for both the formulations for the same magnitude of pitch angle (45.7 degrees). This (Fig. 9) confirms the speculation made in the earlier paragraph. The steady state oscillations are much larger in amplitude for random noise formulation. This probably sheds light on why FLIER is better than the random noise formulation.

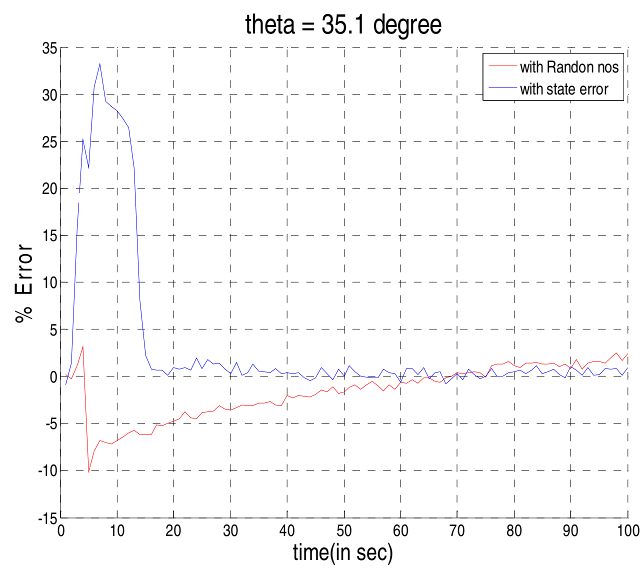

Fig. 7. Effect of driving noise for 35.1 degree pitch angle

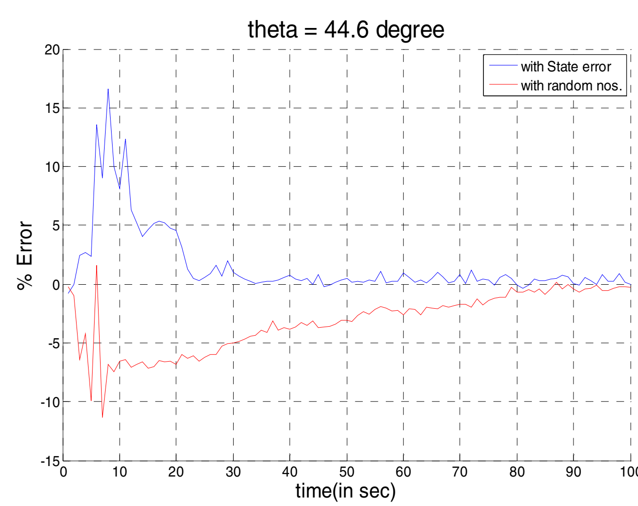

Fig. 8. Effect of driving noise for 44.6 degree pitch angle

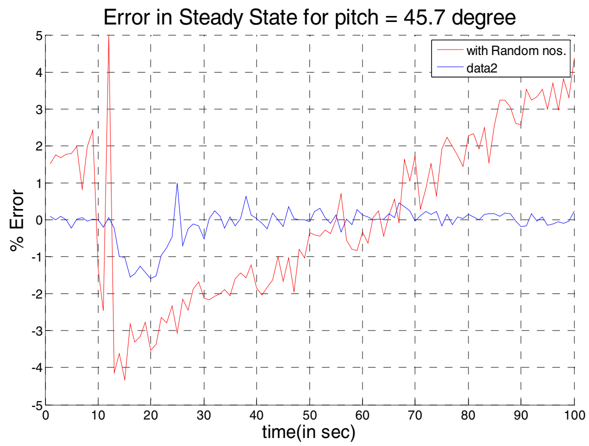

Fig. 9. Effect of driving noise after the filter has converged (magnified steady state comparison)

VIII. Conclusion

Design and real-time hardware implementation of a novel sensor fusion algorithm were presented in this paper with detailed results of hardware-in-loop-simulation. The results showed that the proposed algorithm yields faster convergence with better steady state performance compared to conventional EKF.

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